Heat and mass transfer on MHD flow through a porous medium over a stretching surface with heat source

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ABSTRACT

An attempt has been made to study the heat and mass transfer effect on the flow over a stretching sheet in the presence of a heat source. The novelty of the present study is to consider the span wise variation of magnetic field strength, heat source and heat flux. It is also considered the effect of viscous dissipation. The method of solution involves similarity transformation which leads to an exact solution of velocity field. The coupled non-linear and non-homogeneous heat equation has been solved by applying Kummer’s function. The non-homogeneity of the heat equation is contributed by the consideration of viscous dissipative energy.

KEYWORDS: Heat source, Viscous dissipation, Porous medium, Kummer’s function.

1. INTRODUCTION

Momentum and heat transfer in a boundary layer over a linear stretching sheet have been studied extensively in the recent and past because of its ever-increasing usage in polymer processing industry, in particular in manufacturing process of artificial film and artificial fibers. In some applications of dilute polymer solution, such as the 5.4% solution of polyisobutylene in cetane, the visco-elastic fluid flow occurs over a stretching sheet.

Some of the typical application of such study is polymer sheet extrusion from a dye, glass fiber and paper production, drawing of plastic films etc. A great deal of literature is available on the two-dimensional visco-elastic boundary layer flow over a stretching surface where the velocity of the stretching surface is assumed linearly proportional to the distance from a fixed origin. Flow and heat transfer study over moving smooth surfaces are of immense effect in many technological processes, such as the aerodynamic extrusion of plastic sheet, rolling, purification of molten metal from non-metallic inclusion by applying magnetic field and extrusion in manufacturing processes. In continuous casting, consists of pouring molten metal into a short vertical metal die or mould, which is open at both ends, colling the melt rapidly and withdrawing the solidified product in a continuous length from the bottom of the mould at a rate consistent with that of pouring, the casting solidified before leaving the mould. The mould is cooled by circulating water around it. The process is used for producing blooms, billets and slabs for rolling structural shaped, it is mainly employed for copper, brass, bronze, aluminum and also increasingly with cast iron and steel.

The problem of heat and mass transfer combined with chemical reaction is very important due to its industrial applications. Heat and mass transfer occur simultaneously in processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, other examples of industrial applications are curing of plastic, cleaning and chemical processing of materials relevant to the manufacture of printed circuitry, manufacture of pulp-insulated cables etc. Two types of chemical reaction can take place, homogeneous reaction which occurs uniformly thought given phase, while a heterogeneous reaction takes place in a restricted region or within the boundary of a phase.

The study of two-dimensional boundary layer flow, heat and mass transfer over a Porous stretching surface is very important as it finds many practical applications in different
areas. To be more specific, it may be continuous strips or filaments by drawing them through a quiescent fluid and that in the process of drawing these strips, are sometimes stretched. Viscous dissipation changes the temperature distribution by playing a role like an energy source, which leads to affect heat transfer rates. The merit of the effect of viscous dissipation depends on whether the sheet is being cooled or heated.


Sharma and Singh (2009) studied effects of ohmic heating and viscous dissipation on steady MHD flow near a stagnation point on an isothermal stretching sheet. Kumar (2009) considered radiative heat transfer by hydromagnetic flow and viscous dissipation over a stretching surface in the presence of variable heat flux. The viscous dissipative heat effects on the steady free convection and on combined free and forced convection flows have been extensively studied by Ostrach (1954). The problem of dissipation effects on MHD nonlinear flow and heat transfer past a Porous surface with prescribed heat flux have been studied by Anjali Devi and Ganga (2010). Abo-Eldahab and El Axiz (2005) studied the effect of viscous dissipation and Joule heating on MHD free convection flow past a semi-infinite vertical flat plate with power law variation in surface temperature in the presence of the combined effect of Hall and iso-slips currents. Rajeswari et al (2009) have studied the effect of chemical reaction, heat and mass transfer on nonlinear MHD boundary layer flow through vertical Porous surface with heat source in the presence of suction. Anjali Devi and Ganga (2009) studied effects of viscous and Joules dissipation on MHD flow, heat and mass transfer past a stretching Porous surface embedded in a Porous medium. Recently, the study of heat transfer analysis of the unsteady flow of a Maxwell fluid over a stretching surface in the presence of a heat source/sink has gained considerable attention by Mukhopadhyaya (2012). Singh and Singh (2012) have studied MHD flow with viscous dissipation and chemical reaction over a stretching Porous plate in Porous medium.

The objective of the present analysis is to consider the mass transfer aspect of the work of Hitesh Kumar (2011). Further, we have incorporated the viscous dissipation in the energy equation. In the present study we have considered an electrically conducting fluid where as in the literature of earlier work, the author has restricted to non-conducting fluid.

2. FLOW ANALYSIS

A steady laminar and two dimensional flow of a viscous incompressible electrically conducting flow through porous medium over a stretching surface with heat source and has been considered. In our analysis we have taken x-axis along the wall in the direction of motion of the flow, the y-axis being normal to it and $u$ and $v$ are tangential and normal velocity components respectively. The applied magnetic field is perpendicular to the plate. Here, it is assumed that the induced magnetic field produced by the motion of the electrically
conducting fluid has been neglected. Thus for the problem under consideration, the equations of the laminar boundary layer are,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$  \hspace{1cm} (2.1)

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial x^2} - \frac{u}{K_p'} - \frac{\sigma B_0^2 u}{\rho}$$  \hspace{1cm} (2.2)

The boundary conditions are

$$u = ax, \ v = -v_0 \text{ at } y = 0 \hspace{1cm} \text{as} \ y \to \infty$$  \hspace{1cm} (2.3)

Where $\nu$, kinematics viscosity, $K_p'$, permeability of the porous medium, $B_0$, applied magnetic field, $\rho$, density of the fluid.

In order to solve equations (2.1) and (2.2) with boundary conditions (2.3), the following transformations are introduced

$$u = axF'(\eta), \ v = -\sqrt{\nu a} F(\eta), \ \eta = \frac{\sqrt{\nu}}{\sqrt{\rho} a} y$$  \hspace{1cm} (2.4)

Using (2.4), equation (2.2) gives

$$F''' + FF'' - F' - \left( M^2 + \frac{1}{K_p'} \right) F' = 0$$  \hspace{1cm} (2.5)

With boundary conditions

$$F(0) = \lambda, F'(0) = 1, F'(\infty) = 0$$  \hspace{1cm} (2.6)

Where $\eta$ is the similarity variable, a prime denotes differentiation w.r.t $\eta$, $K_p = K_p' a / \nu$, the permeability parameter, $M^2 = \frac{\sigma B_0^2}{\rho a}$, the magnetic parameter and $\lambda > 0$ for suction at the stretching plate.

The exact solution (2.5) with boundary conditions (2.6) is

$$F(\eta) = \frac{1}{\alpha} \left( \alpha^2 - \left( M^2 + \frac{1}{K_p} \right) e^{-\alpha \eta} \right),$$  \hspace{1cm} (2.7)

Where $\alpha = \frac{\lambda}{\sqrt{\lambda^2 + 4 \left( 1 + M^2 + \frac{1}{K_p} \right)}}$.

The limiting case of expression (2.7) when $K_p \to \infty$ (very large value of $K_p$), yields the result of Tak and Lodha (2005).
3. **SKIN FRICTION**

The non dimensional form of skin friction, $\tau^*$ at the wall is

$$\tau^* = \mu \left( \frac{\partial u}{\partial y} \right)_{\gamma=0} = F^*(0) = -\alpha$$

4. **HEAT TRANSFER ANALYSIS**

By using boundary layer approximations the equation of energy for temperature $T$, in presence of internal heat generation or absorption and neglecting Joule’s dissipation is given by

$$\frac{u \partial T}{\partial x} + v \frac{\partial T}{\partial y} = K \frac{\partial^2 T}{\partial y^2} + \frac{v}{\rho C_p} \left( \frac{\partial u}{\partial y} \right)^2 - S'(T - T_\infty)$$

(4.1)

Where $K$, the thermal conductivity, $C_p$ the specific heat at constant pressure.

With boundary conditions

$$-K \frac{\partial T}{\partial y} = q_w = E_0 \chi^2 \text{ at } y = 0$$

$$T \rightarrow T_\infty \text{ as } y \rightarrow \infty$$

(4.2)

Where $q_w$, the ratio of heat transfer, $E_0$, a positive constant and $T_\infty$, temperature at a large distance from the wall.

Introducing the similarity variable $T - T_\infty = \frac{E_0 \chi^2}{K} \frac{u}{a} \theta(\eta)$ and using (2.4) in (4.1) we get

$$\theta^* + P_r F \theta' - 2P_r F' \theta - S \theta = -E_c P_r F^{*2}$$

(4.3)

$$P_r = \frac{\mu C_p}{K}, \text{ the prandtl number, } S = \frac{S'}{a}, \text{ the source parameter,}$$

$$E_c = \frac{a}{C_p \left( \frac{E_0}{K} \right) \left( \frac{u}{a} \right)} \text{, the Eckert number.}$$

The boundary conditions are

$$\theta' = -1 \text{ at } \eta = 0$$

$$\theta \rightarrow 0 \text{ at } \eta \rightarrow \infty$$

(4.4)

We introduce a new variable $\xi = -\frac{P_r}{\alpha^2} e^{-a\eta}$ and using in (4.2), the equation (4.3), transforms to

$$\xi \frac{d^2 \theta}{d\xi^2} + \left[ \left( 1 - \frac{P_r}{\alpha^2} \left( \alpha^2 - M^2 - \frac{1}{K_p} \right) \right) - \xi \right] \frac{d\theta}{d\xi} + \left( 2 - \frac{S}{\alpha^2 \xi} \right) \theta = -\frac{E_c}{P_r} \alpha^4 \xi$$

(4.5)

With the corresponding boundary conditions
The exact solution of (4.5) subject to the boundary conditions (4.6) can be written in terms of confluent hypergeometric function in terms of similarity variable \( \eta \) and is given as

\[
\theta(\eta) = -\frac{E_1 P_r e^{-2a\eta}}{4 - 2K_1 - S/\alpha^2} + \frac{P_r}{\alpha^2} \left[ \frac{\alpha}{P_r} + \frac{E_1 \alpha^2}{2 - K_1 - S/2\alpha^2} \right] e^{\frac{K_1 + K_2 - 4}{2} \frac{1 + K_1; -\frac{P_r e^{-a\eta}}{\alpha^2}}{1 + K_1; -\frac{P_r e^{-a\eta}}{\alpha^2}}} \\
\left( \frac{K_1 + K_2}{2} \right) F_1 \left( \frac{K_1 + K_2 - 4}{2} ; 1 + K_1 ; -\frac{P_r}{\alpha^2} \right) F_1 \left( \frac{K_1 + K_2 - 2}{2} ; 1 + K_1 ; -\frac{P_r}{\alpha^2} \right)
\]

(4.7)

5. **Mass Transfer Analysis**

The equation for species concentration with chemical reaction is given by

\[
\frac{\partial C}{\partial x} + \nu \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2}
\]

(5.1)

With the boundary conditions

\[
-D \frac{\partial C}{\partial y} = m_w = E_1 x^2 \text{ at } y = 0 \\
C \rightarrow C_\infty \text{ at } y \rightarrow \infty
\]

(5.2)

Introducing the similarity variable \( C - C_\infty = E_1 x^2 \sqrt{\nu a} \phi(\eta) \) and using (2.7), in equation (5.1) we get

\[
\phi'' + S_c F \phi' - S_c F' \phi = 0
\]

(5.3)

where \( C \), the species concentration of the fluid, \( C_\infty \), the species concentration of the fluid away from the wall, \( m_w \), the rate of mass transfer, \( D \), the diffusivity coefficient, \( E_1 \), a positive constant and \( S_c = \frac{D}{D} \), the Schmidt number.

The boundary condition becomes

\[
\phi' = -1 \text{ at } \eta = 0 \\
\phi \rightarrow 0 \text{ at } \eta \rightarrow \infty
\]

(5.4)

Again introducing a new variable \( \zeta = -\frac{S_c}{\alpha^2} e^{-a\eta} \), the equation (5.3) becomes
\[ \zeta \frac{d^2 \varphi}{d \zeta^2} + \left[ 1 - \frac{S_c}{\alpha^2} \left( \alpha^2 - M^2 - \frac{1}{K_p} \right) \right] \frac{d \varphi}{d \zeta} - \zeta \frac{d \varphi}{d \zeta} + 2 \varphi = 0 \]  

(5.5)

The corresponding boundary conditions are

\[ \varphi(\zeta = 0) = 0, \quad \varphi'(\zeta = \frac{S_c}{\alpha^2}) = -\frac{\alpha}{S_c} \]  

(5.6)

The exact solution of equation (5.5) subject to the boundary condition (5.6) is given by

\[ \varphi(\eta) = \frac{e^{-S_c \eta} \Gamma \left( S_1 - 2; 1 + S_1; -\frac{S_c e^{-\alpha \eta}}{\alpha^2} \right)}{\alpha S_1 \Gamma \left( S_1 - 2; 1 + S_1; -\frac{S_c}{\alpha^2} \right)} \left[ \frac{S_c}{\alpha^2} \frac{S_c - 2}{\alpha \left( 1 + S_1 \right)} \Gamma \left( S_1 - 1; 1 + S_1; -\frac{S_c}{\alpha^2} \right) \right] \]  

(5.7)

Where \( S_1 = \frac{S_c}{\alpha^2} \left( \alpha^2 - M^2 - \frac{1}{K_p} \right) \)

6. RESULT AND DISCUSSION

The momentum, heat and mass transfer equations are characterized by the magnetic parameter \( M \), permeability parameter \( K_p \), heat source parameter \( S \), Suction parameter \( \lambda \), Prandtl number \( \Pr \), Eckert number \( E_c \) and Schmidt number \( S_c \).

Fig.1 shows the transverse velocity distribution. The variation of transverse velocity is confined within a few layers near the plate. It is observed that magnetic parameter and permeability parameter decelerate the transverse velocity and the suction parameter accelerate it. Therefore, presence of porous matrix and magnetic interaction parameter has counter productive and suction has beneficial roles on transverse velocity. It is further noted tat this component of velocity attains stability within a short span along the flow direction. The present study is in good agreement with the result of Hitesh kumar (2011).

Fig.2 shows the variation of longitudinal velocity in the flow domain. It is interesting to note that an increase in suction parameter, Magnetic parameter and presence/absence of porous matrix reduce the longitudinal velocity at all points. In particular, in the absence of magnetic field \( M=0 \) the velocity attends the lowest value in both the cases i.e. with or without porous matrix. Therefore, it is suggested that presence of suction and magnetic interaction fails to contribute to accelerate the longitudinal component and hence accelerate this process of attainment of asymptotic value. It is faster in case of \( M=0 \) i.e. complete absence of magnetic field.

The heat equation, related to the present study is subject to non-homogenous boundary condition with variable temperature gradient, admits similarity solution. Then reduced ordinary differential has been solved in terms of Kummers function.

From fig.3 it is observed that an increase in suction parameter, magnetic parameter, Prandtl number and heat source reduce the temperature at all points but permeability of the medium and Eckert number enhance it. One striking feature of the temperature field is that the temperature increases under the influence of magnetic field in the absence of porous matrix. Therefore, it is imperative to conclude that viscous dissipation energy coupled with resistance
offered by the porous matrix are beneficial for the rise in temperature of the fluid. Further, magnetic interactions alone without porous matrix contribute the rise in temperature.

Fig.4 shows the concentration variation. The Magnetic field enhances the concentration level in both porous and non-porous medium, but Schmidt number and suction parameter reduce it. It is interesting to record that difference in concentration distribution occurs for both porous and non-porous medium. The concentration level increases in the porous medium (Curve I and II).

Now let us discuss the effects of parameters on the skin friction. It is observed that skin friction assumes negative values for all the parameters. Further, an increase in suction parameter in the absence/presence of magnetic field as well as porous matrix decreases the skin friction coefficient. This shows that stronger suction coupled with magnetic interaction leads to a favorable condition in reducing the skin friction which is desirable as because stretching requires less effort.

7. CONCLUSION

- The interaction of magnetic field is proved to be counter productive in enhancing velocity and concentration distribution but beneficial in attaining higher temperature within flow field.
- Presence of suction fails to contribute to accelerate the longitudinal component.
- Inclusion of viscous dissipation in a flow through porous media is beneficial for gaining temperature.
- Difference in concentration distribution occurs for both porous and non-porous medium.
- Stronger suction compelled with magnetic field interaction reduces the skin friction coefficient which is a desirable condition for stretching.

REFERENCES


Sharma, P.R., Singh, G. “Effects of ohmic heating and viscous dissipation on steady MHD flow near a stagnation point on isothermal stretching sheet”. Thermal Science, 13(1). PP. 5-12 (2009).


Fig. 3. Temperature Profile

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Fig. 4. Concentration Profile

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